TEMPERATURE RISE TEST ON POWER TRANSFORMERS

Numerical Method for Calculating the Average Resistance of a Winding at Shut Down Based on the Cooling Curve Method.

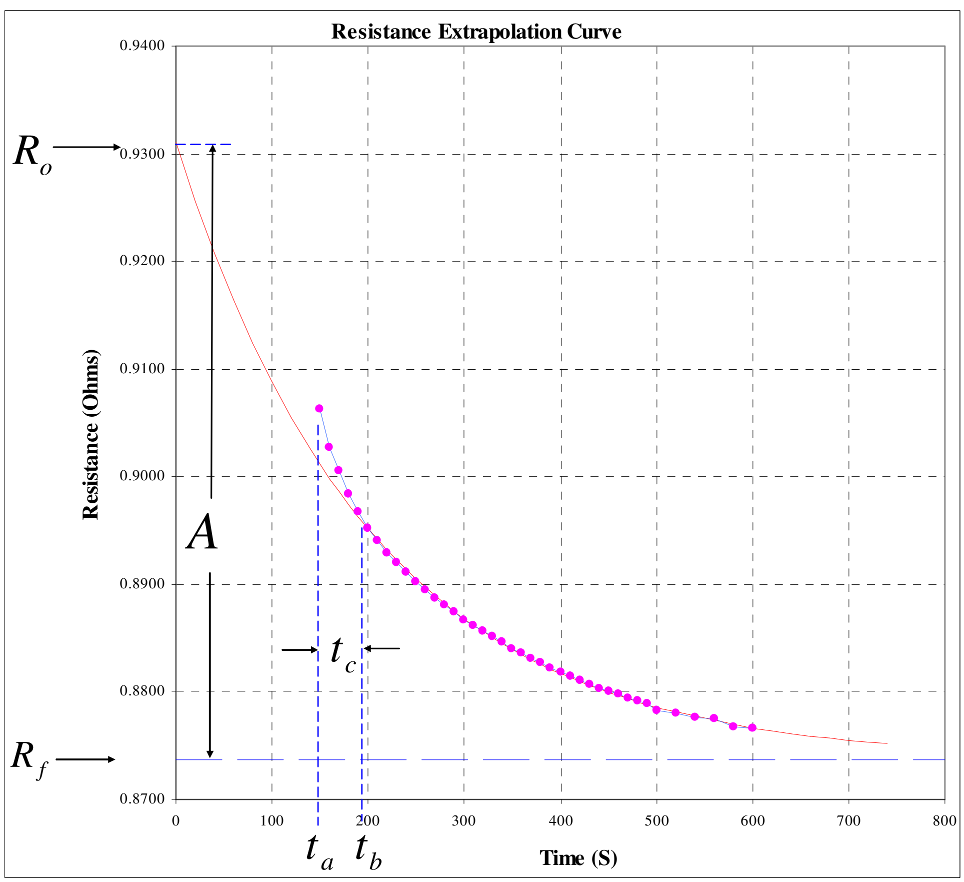
by Bertrand Poulin, October 2009

At the end of a temperature rise test on a power transformer, a series of resistance measurements is made on one or more windings in order to determine the average temperature of windings. Typically, a series of 10 to 30 measurements are made within 10 minutes after shutting down the power. The time elapsed from shut down is also noted for each measurement. Next, the resistances values are plotted against time on a graph with suitable scaling and a curve is fitted through the points and extrapolated back to determine the resistance at time zero, the moment the power was shut down. Nowadays, this process is automated using dedicated software to make it more accurate and less subjective. This document deals with one numerical method to accomplish the extrapolation back to shut down time.

It is assumed that from the time of shut down, the winding cools down exponentially from its initial temperature to a final temperature related to the temperature of the oil surrounding the winding. The temperature of the oil is assumed approximately constant for the period of time the measurements are made. This is true if the cooling time of the oil to the ambient air is much greater than the cooling time of the winding to the oil.

We start with a set of n values of resistance noted R_1, R_2, ..., R_n and n corresponding values of time noted t_1, t_2,..., t_n on a graph as seen in figure below. We want to fit an exponential curve to these points.

R_0 = Resistance at shut down
R_f = Asymptote to the curve
$A = R_0 - R_f$
t_a = time to first reading
t_c = time for current to build
t_b = time to first valid reading



For the curve fitting process, we use the following expression:

$$R = (R_0 - R_f) \bullet e^{-t/\tau} + R_f \qquad (1)$$

Let us make $A = R_0 - R_f$ and $m = 1/\tau$. We obtain

$$R = A \bullet e^{-mt} + R_f \quad \text{or} \quad R - R_f = A \bullet e^{-mt} \qquad (2)$$

Three parameters are required to completely determine the curve: A, m and R_f.

Taking the natural logarithm of eq. (2), we obtain

$$\ln(R - R_f) = \ln A - mt$$

This expression is a straight line. Therefore, a linear regression process can be used to determine the slope (m) and the intercept (lnA) of this line. For that, R_f need to be determined first. Many algorithms in use throughout the industry use an iterative process to determine R_f. In short, they find the value of R_f which provides the best fit of the theoretical exponential curve through the measured points. This is either accomplished numerically by an automated process or by manually selecting R_f and looking at the results of a linear regression process in a spreadsheet, in a trial and error process.

It is our opinion that there is a better way to determine R_f than this iterative or trial and error method. By taking the time derivative of equation 2 above, we have:

$$\frac{\partial R}{\partial t} = -m \bullet A \bullet e^{-mt} \qquad (3)$$

Combining equations (2) and (3), we obtain

$$\frac{\partial R}{\partial t} = -m \bullet (R - R_f) \qquad \frac{\partial R}{\partial t} = -mR + mR_f \qquad (4)$$

$$\frac{\Delta R}{\Delta t} = -mR + mR_f \qquad (5)$$

This expression is also a straight line of slope -m and intercept mR_f. By taking the numerical derivative of R versus time for a number of points, we can apply a linear regression to obtain R_f.

The overall process is as follows. From the original set of data $(t, R)_i$, we calculate a new set of data $(x, y)_j$. It is well known that numerical differentiation amplifies noise present in digital data sets. For better performance, some form of noise reduction must be used when noise is expected to be superimposed. In this algorithm, the following definition are used:

$$x_j = R_{j-2} + R_{j-1} + R_j + R_{j+1} + R_{j+2}$$

$$Y_j = \left(\frac{\Delta R}{\Delta t}\right)_j = \frac{R_{j+2} - R_{j-2}}{t_{j+2} - t_{j-2}}$$

for j = 3, 4, ..., n-2

This effectively applies averaging to the calculation of the derivative. With this data, a first linear regression is performed using the following well known expressions.

n = number of points used in the regression

$y = mx + b$ (standard equation for a straight line)

$$m = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$ = slope of the line

$$b = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}$$ = intercept of he line

$$R_f = -\frac{b}{m}$$

Once R_f is determined, a second set of data is computed and a second linear regression is made using the following equations.

$$x_i = t_i$$

$$y_i = \ln(R_i - R_f)$$

$$A = e^b$$

$$R_0 = A + R_f$$

This numerical method has been implemented successfully since 1981 and has been tested on several hundreds of cases. It is actually a numerical equivalent of a graphical method described in IEC 76-2 1993, appendix C.